The set of rational numbers is generally denoted by an upper case \( \mathbb{Q} \) (which may seem strange, but there is a rational explanation 😊). Any rational number can be written as a fraction. To get the decimal form of the fraction, we perform a division. The answer to a division is called the quotient.

The Italian mathematician Giuseppe Peano (1858-1932) used the \( \mathbb{Q} \) notation in 1895 to represent the set of rational numbers because the Italian word for ‘quotient’ is ‘quoziente’. Peano was the founder of mathematical set theory (we have used some set notations in describing number systems) and mathematical logic.

In logic, one places a bar over a letter in order to signify the opposite. For example, if ‘a’ represents ‘true’, then ‘\( \bar{a} \)’ means ‘not true’ (i.e. false). It follows, therefore, that if \( \mathbb{Q} \) represents the set of rational numbers, then \( \bar{\mathbb{Q}} \) means ‘not rational’ (i.e. irrational).

Let’s review some basic facts about numbers and see where fractions sit in this hierarchy. Up until grade 10 youngsters use the Real Number System, which is represented by the letter \( \mathbb{R} \). Real numbers fall into two classifications: rational numbers (represented by \( \mathbb{Q} \)) and irrational numbers (represented by \( \bar{\mathbb{Q}} \), and pronounced “Q-bar”). In mathematical set notation, we would write \( \mathbb{R} = \mathbb{Q} \cup \bar{\mathbb{Q}} \) (i.e. the set of real numbers consists of the union of the sets of rational numbers and irrational numbers). Note: a more detailed explanation of the math symbols used in this article appear just before the ‘References’ section at the end of this article.

Rational Number: any real number that can be expressed in the form \( \frac{a}{b} \) where ‘a’ and ‘b’ are integers, but ‘b’ cannot equal zero (\( b \neq 0 \)). ‘Integer’ is explained shortly. Zero is never permitted to be the denominator of a fraction because division by zero is impossible.

These numbers are rational numbers: \( 8.2, \frac{2}{3}, \sqrt{25} \).

May 30, 2014 -- Today our youngsters seem to have a lot of trouble with fractions. The advent of electronic calculators is largely to blame for this. Students rely on calculators for all types of arithmetic calculations – even basic addition. Great grandparents will recall when the mathematics curriculum included the processes (or algorithms) for long division and square root. The latter has disappeared altogether from our schools, and the former may be taught but fades from memory as reliance upon electronic devices increases.

A youngster presented with the calculation 248 ÷ 17 would type it into a calculator, getting an answer such as this on the display: 14.58824

Does the child know if this is an exact answer or an approximate answer (which has been rounded off by the machine)? Probably not. For day-to-day computations in business, shopping or measuring, it is not necessary to know the answer to anything more than a couple of decimal places. Scientists, mathematicians and engineers, however, need to know more.
(a) \(8.2 = \frac{8}{10} = \frac{8}{1} \times \frac{1}{5} = \frac{41}{5}\), which is in the form \(\frac{a}{b}\), with \(a=41\) and \(b=5\)

(b) \(\frac{2}{3}\) is already in the form \(\frac{a}{b}\), with \(a=2\) and \(b=3\)

(c) \(\sqrt{25} = 5 = \frac{5}{1}\), which is in the form \(\frac{a}{b}\), with \(a=5\) and \(b=1\)

A rational number is a fraction. The numerator (the top) and the denominator (the bottom) are integers. A proper fraction, such as \(\frac{1}{8}\), has a numerator that is smaller than the denominator, and represents ‘a part of a whole’ (e.g. one pizza slice in the diagram is one-eighth of the whole pizza).

An integer itself, such as \(38\), is a rational number because we can always rewrite the number in a fractional form with a denominator of 1 (i.e. \(38 = \frac{38}{1}\)). Such a fraction is called an improper fraction because the numerator is larger than the denominator.

When a rational number is converted into its equivalent decimal form, the decimal part of the number will always either terminate or repeat.

Examples:

(a) 8.2 is already in decimal form. The decimal part terminates (or ends). There are no other digits after the 2 (you could think that there are an infinite number of zeros after the 2, in which case the decimal part ends in a set of repeating zeros. Either way of thinking still fits the pattern for rational numbers in decimal form).

(b) \(\frac{2}{3} = 0.6666666666… \approx 0.\overline{6}\) The decimal part repeats forever. The notation \(0.\overline{6}\) is a shorthand form that indicates that the 6 repeats.

(c) \(\sqrt{25} = \frac{5}{1} = 5.0\) The decimal part terminates – or can be thought of as a set of infinitely repeating zeros.

(d) \(\frac{3}{4} = 0.75\), a terminating decimal.

(e) \(\frac{24}{7} = 3.428571428571428… = 3.4\overline{28571}\) The decimal part has six infinitely repeating decimals.
The decimal part has two infinitely repeating decimals.

**Irrational Number:** any real number that cannot be expressed in the form \( \frac{a}{b} \) where ‘a’ and ‘b’ are integers, \( b \neq 0 \). For example, any square root that does not work out evenly, such as \( \sqrt{13} \), is an irrational number.

The famous number \( \pi \), represented by \( \pi \), is another example of an irrational number (see image). Such numbers have decimal forms that never terminate and never end in a repeated pattern.

Examples:
1. \( \sqrt{13} = 3.605551275469\ldots \) (no repeating pattern ever)
2. \( \pi = 3.141592653897\ldots \) (no repeating pattern ever)

The set of rational numbers includes the set of integers. These are numbers that can be written without a fractional part. They may be positive or negative. The capital letter \( \mathbb{Z} \) is usually used to represent the set of integers. Mathematically, we would write:

\[ \mathbb{Z} = \{ \ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots \} \]

The series of dots at each end shows that the set continues infinitely in each direction (into the negatives, to the left, and into the positives, to the right). The \( \mathbb{Z} \) notation is from the German word ‘zahlen’ which means ‘numbers’.

As we said, a rational number can be expressed as a fraction \( \frac{a}{b} \) where ‘a’ and ‘b’ are integers, \( b \neq 0 \). Since an integer may be positive or negative, it is worth reviewing the rules for the division of signed numbers:

- Positive ÷ Positive = Positive
- Positive ÷ Negative = Negative
- Negative ÷ Positive = Negative
- Negative ÷ Negative = Positive

Thus, the positive number \( \frac{4}{5} \) is equivalent to \( \frac{4}{5} \) and \( \frac{-4}{-5} \).

The negative number \( -\frac{12}{13} \) is equivalent to \( -\frac{12}{13} \) and \( \frac{12}{-13} \).

The set of integers includes three types of numbers: the natural numbers, zero, and the opposite (or negatives) of the natural numbers. Natural numbers are often described as being the “counting numbers”.

\[ \mathbb{N} = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots \} \]
If the number zero is added to the set, mathematicians give the set a new name – the set of Whole Numbers.

\[ W = \{ 0 \} \cup \mathbb{N} = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots \} \]

The set of integers, once again, is

\[ \mathbb{Z} = \{ \ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots \} \]

As we grow up, we tend to learn about numbers from the simplest sets to the more complicated. First, we learn to count – the natural numbers. Then we learn what ‘nothing’ means, and that the digit ‘0’ symbolizes zero. We have advanced to the set of whole numbers. It is likely that we stay with positive numbers for a while, learning about positive fractions next – part of the rational numbers. The concept of negative numbers is soon introduced, and by the time students graduate from elementary school they have been introduced to the complete set of rational numbers. Further studies in high school will bring in irrational numbers, completing the set of real numbers, and once the topic of solving quadratic equations is encountered a new type of number, an imaginary number, will be introduced.

![Complex Numbers Diagram](image)

When real numbers and imaginary numbers are combined together in one set of numbers, the resultant set is called the complex numbers.

**Complex Numbers**

\[ \text{Complex Numbers} = \text{(Real Numbers)} \cup \text{(Imaginary Numbers)} \]

Complex numbers may be a future topic of discussion. This article is focusing on fractions, so explanations will not be given on the topic of complex numbers at this time.

Giuseppe Peano, the Italian mathematician mentioned earlier, played a key role in number theory and set theory. Venn Diagrams are used in set theory to illustrate how different groups of objects are related. A Venn Diagram involves loops: each loop represents a set of objects that has a common characteristic. Loops may intersect with other sets, as shown below:

![Venn Diagram](image)

There are two loops in this diagram – one contains the set of “garden flowers” and the other contains the set of “red objects”. Obviously, there are red garden flowers – and that set is represented by the yellow region which is the intersection of the two loops.

Venn Diagrams are a convenient way to illustrate how sets are interrelated. Loops which are contained within loops are called subsets. For example, the set of natural numbers is a subset of the set of whole numbers.
Earlier I included a chart that illustrated the hierarchy of various number systems. The same relationships can be expressed using a Venn diagram:

![Venn Diagram of Number Systems]

**Long Division**

A fraction such as \( \frac{a}{b} \) may be interpreted as \( a \div b \), a division. To convert a fraction into its equivalent decimal form, the denominator is divided into the numerator. Let's review the process of long division as we convert the fraction \( \frac{3}{4} \) into its decimal form.

The division is set up as follows:

\[
4 \overline{)3.0}
\]

1. Can I divide 4 into 3? No. Add a decimal point after the 3 and add a zero, making the problem look like this:

\[
4 \overline{)3.0}
\]

2. Can I divide 4 into 30? Yes. It goes in 7 times (4×7=28), and there would be 2 left over (this remainder must be less than the number you are dividing by, which in this case is 4). The 7 is written in the answer section on top of the horizontal bar, over the zero of the 30. The decimal point in the answer section sits directly over the decimal point of the number underneath it. The problem now looks like this:

\[
4 \overline{)3.0}
\]

3. 4×7=28, so the 28 is written under the 30 (we can ignore the decimal point from now on). 30 – 28 = 2, and this answer from the subtraction is written at the bottom.

\[
\begin{array}{c|c|c|c|c}
& & & 7 & \hline
\hline
4 & \overline{)3.0} & -28 & \hline
7 & 2 & \hline
\end{array}
\]
4. Can I divide 4 into 2? No. Add another zero to the string of decimals after 3.0, making it 3.00 – and add another zero to the 2 at the bottom (making it 20).

\[
\begin{array}{c}
4 \overline{)3.00} \\
\underline{28} \\
\underline{20} \\
\end{array}
\]

5. Can I divide 4 into 20? Yes. It goes in 5 times (4X5=20), and there is nothing remaining. The 5 is added to the answer section.

\[
\begin{array}{c}
4 \overline{)3.00} \\
\underline{28} \\
\underline{20} \\
\end{array}
\]

6. 4X5=20, so the 20 is written under the 20 at the bottom. 20-20=0, so the remainder 0 is written under the subtraction line.

\[
\begin{array}{c}
4 \overline{)3.00} \\
\underline{28} \\
\underline{20} \\
\underline{0} \\
\end{array}
\]

7. Because the remainder is now zero, the division process has ended. The fraction \( \frac{3}{4} \) has been converted into 0.75, its decimal form. If the remainder was anything other than zero, we could continue the process followed in steps (3), (4), and (5). Each cycle through these steps adds one more digit to the decimal answer. If the decimal does not terminate, then – eventually – you would discover that you would get a repeating set of digits in the decimals.

If you wanted to work out 3÷4 on a calculator, you would turn the calculator on, press the “3” key, followed by the “÷” key and the “4” key, and then you press “=“ or the ENTER key. The display would now show the answer: 0.75
Try to convert $\frac{24}{7}$ into its equivalent decimal form by using long division. After giving it your best shot, compare your solution to the one at the right.

You will notice that the string of decimals after the decimal point begins with 428571, and then the pattern begins again. The decimal portion of this number will continue forever without ending, but it will do so as a repeating string of digits.

What if this question had been worked out on a calculator? Typing in $24 \div 7$ would have produced an answer of something like 3.4285714, depending on how many digits the calculator is set up to display. Calculators do NOT produce exact answers: if the decimals would run off the display screen, calculators will round off the answer.

The answer is $3.428571428571428\ldots$

1 2 3 4 5 6 7 8 9 10 \(\leftarrow\) digit position

If the calculator could display 8 digits of an answer, it would examine the 9\(^{th}\) digit (in this case a “2”). Since it is less than 5, the remainder of the answer is simply “chopped off”, leaving the display as 3.4285714.

If the calculator could display 9 digits of an answer, it would examine the 10\(^{th}\) digit (in this case an “8”). Since it is 5 or more, the 9\(^{th}\) digit would get rounded up from 2 to 3 – and then the remainder of the answer would be discarded, leaving the display as 3.42857143

In either case, the calculator’s answer is an approximation to the correct answer: it is not the exact answer. This type of rounding or chopping (the ‘official’ term is truncation) occurs for long decimal answers.

Very large numbers, the results of calculations such as $92,568 \times 997,662$, also pose problems for calculator displays. Let us once again suppose that our calculator can only display a maximum of 8 digits.

$92,568 \times 997,662 = 92,351,576,016$

The exact answer has 11 digits – 92 billion, 351 million, 576 thousand, 16. The calculator can’t simply show you the 8 digits 92351576 because that would mean 92 million instead of 92 billion. What do calculators do in such cases? They display the answer in a different form, called exponential notation. The explanation follows…

The Decimal System of numbers is based on the fact that each position in a number represents a power of ten. Consider the image below:
The digits to the LEFT of the decimal point occupy positions of ever-increasing size.
The ‘Units’ position = $10^0 = 1$ In the number above, there are no units: $0 \times 10^0$
The ‘Tens’ position = $10^1 = 10$ In the number above, there is 1 ten: $1 \times 10^1$
The ‘Hundreds’ position = $10^2 = 100$ The number above has 7 hundreds: $7 \times 10^2$
The ‘Thousands’ position = $10^3 = 1000$ The number above has 6 thousands: $6 \times 10^3$

The digits to the RIGHT of the decimal point occupy positions of ever-decreasing size.
The ‘Tenths’ position = $10^{-1} = \frac{1}{10}$ The digit 3 indicates the number has $\frac{3}{10}$
The ‘Hundredths’ position = $10^{-2} = \frac{1}{100}$ The digit 2 indicates the number has $\frac{2}{100}$
The ‘Thousandths’ position = $10^{-3} = \frac{1}{1000}$ The digit 8 indicates the number has $\frac{8}{1000}$

A number such as 24000 could be written as $24 \times 1000$ or $24 \times 10^3$. This latter representation involves multiplying a number by a power of ten. Other equivalent forms for 24000 are $2.4 \times 10^4$ and $0.24 \times 10^5$. The exponent that is on the ‘ten’ indicates the number of places that the decimal point would have to be moved to the right:

- $24 \times 10^3$ means $24 \times 1000 = 24000$ (the decimal point was moved from being immediately after the 4, three positions to the right – adding three zeros).
- $2.4 \times 10^4$ means $2.4 \times 10000 = 24000$ (the decimal point was moved from its position between the 2 and 4, four positions to the right – skipping over the 4 and adding three zeros).
- $0.24 \times 10^5$ means $0.24 \times 100000 = 24000$ (the decimal point was moved from its position in front of the 2, five positions to the right – skipping over the 2, 4 and adding three zeros).

A calculator cannot display a power of ten in forms such as $10^3$ or $10^4$. It uses “exponential notation,” showing E03 and E04 instead (the “E” indicating that the following number is to be interpreted as the exponent for 10.)

Thus, a very large number such as 92,351,576,016 might be displayed as something like 9.2351576E10 or 0.92352E11 on a calculator, using exponential notation. Notice that, once again, the calculator’s answer is not an exact answer, but an approximation to the exact answer.
Converting Improper Fractions into Mixed Numbers

Next month’s article is about something called continuous fractions – an enrichment topic (one that is not a part of the regular curriculum) that is more advanced than working with basic fractions, but definitely more interesting. Not much fraction skill is required, other than the need to convert an improper fraction (the numerator is larger than the denominator) into a mixed number.

Consider the example shown here: \( \frac{7}{5} \)

People who are proficient at division can do this in their head: 5 goes into 7 once, with 2 left over – so the answer is \( 1 \frac{2}{5} \). If one’s division skills are lacking, the calculator can be a useful tool to assist in this task. There are many techniques for doing this conversion with the aid of a calculator. We’ll look at two of them.

Typing \( 7 \div 5 \) into a calculator will produce the answer 1.4 which we can use to get the fraction form of the answer. Here are two ways to do this:

1. Use our knowledge of ‘place value’. The decimal part of the answer is ‘4’. The 4 is occupying the ‘tenths’ location, so the meaning of 1.4 is \( \frac{14}{10} = \frac{7}{5} \) (after we reduce the fraction into lowest terms).

2. 7 divided by 5 is 1.4, so the whole number part of the mixed number is 1. Take the decimal part of the calculator’s answer (which is 0.4) and multiply it by the denominator 5. The answer is 2. That answer becomes the numerator for the fractional part of the mixed number; the denominator is the 5 (refer to the box at the right for the explanation of why this method works).

Let’s try another: convert the improper fraction \( \frac{45}{8} \) into a mixed number.

Using a calculator, \( 45 \div 8 = 5.625 \)

1. Using place value, the last digit is in the thousandth place. Thus, \( 5.625 = \frac{625}{1000} = \frac{5}{8} \) (after reducing the fraction).

2. Using the calculator, while the answer 5.625 is still on the display, press the subtraction key followed by the number 5, and then “=” . This leaves us with the decimal part of the answer: 0.625. Then press the multiplication key followed by 8 and “=” : you get 5. Thus,

\[
\frac{45}{8} = 5 + 0.625 = 5 + \frac{625 \times 8}{1 \times 8} = 5 + \frac{5}{8} = \frac{5}{8}
\]

Here’s a tough one: convert \( \frac{156}{17} \) into a mixed number.

Using a calculator, \( 156 \div 17 = 9.176470588 \)
Is this exact? Unlikely. The decimal has been rounded or truncated. So the decimal part of the answer doesn’t end (the repeating part isn’t even clear to us). Method #1 won’t work here, but method #2 will.

With the answer still on the display, press the subtraction key followed by 9 and “=”. Only the fraction part of the answer will be displayed. Now press the multiplication key followed by 17 and “=”. You get 3.

Thus, \( \frac{156}{17} = 9\frac{3}{17} \)

To review how to add, subtract, multiply and divide with fractions, refer to the “Reference” section following this article.

This article is the third of a series of mathematics articles published by CHASA.

1. Marvellous Mathematics – Introduction
2. Euclidian Geometry – Article # 1
3. Non-Euclidean Geometry – Article #2

CHASA has received many communications from concerned parents about the difficulties their children are having with the math curriculum in their schools as well as their own frustration in trying to understand the concepts - so that they can help their children.

The intent of these articles is to not only help explain specific areas of history, concepts and topics in mathematics, but to also show the beauty and majesty of the subject.

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Math Symbols and Terms

Set – a collection of things. Brace brackets (or curly brackets) are used to enclose the items, or elements, in a set. A single letter, usually an upper case letter, is traditionally used to name a set. For example, \( A = \{ \text{dog, cat, hamster, rabbit} \} \).

Subset – part of a set. If \( B = \{ \text{dog, cat} \} \), then \( B \) is a subset of \( A = \{ \text{dog, cat, hamster, rabbit} \} \). When represented by a Venn diagram, the loop for a subset appears inside the outer loop.

Operations on sets – union and intersection are two operations that were used in this article. Union is represented by \( \cup \) and intersection is represented by \( \cap \).

If \( C = \{ \text{apple, pear, banana} \} \) and \( D = \{ \text{orange, apple} \} \), then \( C \cap D = \{ \text{apple} \} \) \( \leftarrow \) only elements that are common to both sets

Venn diagram – a picture used to illustrate relationships among sets (as shown above).

Logic – is the study of valid reasoning. The truth and falsity of statements can be determined by following established rules and valid procedures. Mathematical logic uses symbols and special operations, similar to algebra. For example, if ‘a’ represents ‘true’, then ‘\( \bar{a} \)’ means ‘not true’ (i.e. false).

Decimal System – the system of expressing numbers using ‘base 10’. Each digit in a number occupies a position that represents a ‘power of ten’. In base 10, there are ten digits (0,1,2,3,4,5,6,7,8,9). For example, the number 852.7 means...

\[
8 \times 10^2 + 5 \times 10^1 + 2 \times 10^0 + 7 \times 10^{-1} \\
= 8 \times 100 + 5 \times 10 + 2 \times 1 + 7 \times \frac{1}{10} \\
= 800 + 50 + 2 + 0.7
\]

Numbers may be written in other number systems. For example, computers tend to use the Binary System which uses ‘base 2’. There are just two digits in this system: 0 and 1. Each digit in a number represents a ‘power of two’. For example, the number 101.1 means...

\[
1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} \\
= 1 \times 4 + 0 \times 2 + 1 \times 1 + 1 \times \frac{1}{2} \\
= 4 + 0 + 1 + 0.5 \\
= 5.5 \text{ (as a base ten number) }
\]
Natural Numbers – the counting numbers
\[ N = \{ 1,2,3,4,5,6,7,8,9,10,11,\ldots \} \]

Whole Numbers – the counting numbers, plus zero
\[ W = \{ 0 \} \cup N = \{ 0,1,2,3,4,5,6,7,8,9,10,\ldots \} \]

Integers – the counting numbers, zero, and the negatives of the counting numbers
\[ Z = \{ \ldots,-5,-4,-3,-2,-1,0,1,2,3,4,5,\ldots \} \]

Rational Numbers -- any real number that can be expressed in the form \( \frac{a}{b} \) where ‘a’ and ‘b’ are integers, but ‘b’ cannot equal zero. When the fractional form of a rational number is converted into decimal form, the decimal will either terminate or repeat. The set of rational numbers is represented by \( \mathbb{Q} \).

Irrational Numbers -- any real number that cannot be expressed in the form \( \frac{a}{b} \) where ‘a’ and ‘b’ are integers, but ‘b’ cannot equal zero. When the fractional form of an irrational number is converted into decimal form, the decimal will neither terminate nor repeat. The set of irrational numbers is represented by \( \mathbb{Q}' \).

Real Numbers – the union of the sets of rational and irrational numbers
\[ \mathbb{R} = \mathbb{Q} \cup \mathbb{Q}' \]

Imaginary Numbers -- a number that can be written as a real number multiplied by the imaginary unit \( i \), which is defined by its property \( i^2 = -1 \).

Complex Numbers – the union of the sets of real and imaginary numbers. Complex numbers can be expressed in the form \( a + bi \), where \( a \) and \( b \) are real numbers and \( i \) is the imaginary unit, which satisfies the equation \( i^2 = -1 \). In this expression, \( a \) is the real part and \( b \) is the imaginary part of the complex number.

Scientific Notation -- numbers are written in the form \( a \times 10^b \) (‘a’ times ten raised to the power of ‘b’), where the exponent ‘b’ is an integer, and the number ‘a’ is any real number. For example, the number 483 could be written as 4.83 \times 10^2 (meaning 4.83 X 100).

Calculator E-Notation – since most calculators cannot display a complete power of ten such as \( 10^2 \), the letter E followed by the exponent is used. 4.83 \times 10^2 would be displayed as 4.83E2.

Proper Fraction – the numerator (top) is smaller than the denominator (bottom). Example: \( \frac{7}{8} \)

Improper Fraction – the numerator is larger than the denominator. Such fractions can be expressed as mixed numbers, with an integer part and a proper fraction part. Example: \( \frac{19}{4} = 4 \frac{3}{4} \)

Long Division – the process of division performed by hand to determine the answer to a division question such as 546\( \div \)31. 31 is called the ‘divisor’ and 546 is called the ‘dividend’. The answer above the line (17) is called the ‘quotient’ and the number at the bottom (19) is called the ‘remainder’. If the
division proceeded further in order to get decimals, the answer here would be
546÷31=17.61258064516125806451… (a repeating decimal, in this case)

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Converting One-Digit Repeating Decimals into Fractions – (Advanced) Video
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Simplifying Fractions – Math Is Fun

Reducing Fractions – S.O.S. Math
Reducing Fractions – Video (MathPlayGround)
Adding Fractions – Video
Adding and Subtracting Fractions -- text

Subtracting Fractions – Math Is Fun

Multiplying Fractions – Video

Dividing Fractions – Video

Converting Improper Fractions to Mixed Numbers – Video

Set Theory – Math Is Fun

Venn Diagrams -- PurpleMath

Scientific Notation – related to the calculator’s E-notation

Scientific Notation – advanced material